## The Analysis of Two-Way Functional Data Using Two-Way Regularized Singular Value Decompositions

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## Abstract

Two-way functional data consist of a data matrix whose row and column domains are both structured, for example, temporally or spatially, as when the data are time series collected at different locations in space. We extend one-way functional principal component analysis (PCA) to two-way functional data by introducing regularization of both left and right singular vectors in the singular value decomposition (SVD) of the data matrix. We focus on a penalization approach and solve the non-trivial problem of constructing proper two-way penalties from one-way regression penalties. We introduce conditional cross-validated smoothing parameter selection whereby left-singular vectors are cross-validated conditional on right-singular vectors, and vice versa. The concept can be realized as part of an alternating optimization algorithm. In addition to the penalization approach, we brie?y consider two-way regularization with basis expansion. The proposed methods are illustrated with simulation studies and real data examples.

Keywords: Functional data analysis, penalization, regularization, spatial-temporal model ling, basis expansion